Parametric Functions and Substitution

Our sun is an active star that ejects a constant stream of particles into space called the 'solar wind'. From time to time, magnetic activity on its surface also launches fast-moving clouds of plasma into space called 'coronal mass ejections' or CMEs.

When some of these clouds directed at Earth arrive after traveling 93 million miles (150 million km), they cause intense disturbances in Earth's magnetic field. Since the 1800's, these disturbances have been called 'magnetic storms', because instruments on Earth can measure the strength of these disturbances, and they resemble storms in an otherwise very calm magnetic field.

Scientists measure the strength of these magnetic storms in terms of the size of the change they make in the Earth's magnetic field. The strength of Earth's field at the ground is about 0.7 Gauss or 70,000 nanoTeslas. The most intense magnetic storms can change the ground-level field by several percent.

According to research by V. Yurchyshyn, H. Wang and V. Abramenko, which was published in 2004 in the journal Space Weather (vol. 2) the relationship between the magnetic field disturbance, Dst and the Z-component of the interplanetary magnetic field, B_z, is given by:

$$(1) \qquad Dst = -2.846 + 6.54\, B_z - 0.118 B_z^2 - 0.002 B_z^3$$

where Dst and Bz are measured in nanoTeslas (nT).

In 2004, W. D. Gonzales and his colleagues published a paper in the Journal of Atmospheric and Solar Terrestrial Physics, in which they determined a relation between the speed of a solar coronal mass ejection V, in km/sec, and the strength of Dst in nT according to

$$(2) \qquad Dst = 0.00052 \times (0.22\, V + 340)^2$$

The relationship between the travel time to Earth from the sun and the speed of the CME was determined from catalogs of CME events by M. J. Owens and P. J. Cargill in research published in 2002 in the Journal of Geophysical Research (vol. 107, p. 1050) in terms of the transit time in days, T, for these coronal mass ejections and their speed, V, in km./sec by

$$(3) \qquad T = -0.0042 \times V + 5.14$$

They also found that the maximum interplanetary magnetic field strength of the CME was given by

$$(4) \qquad B_T = 0.047\, V + 0.644$$

1) From the equation 2 and 3 above, find a function that gives Dst in terms of the transit time of the CME. Write the result in expanded form as a quadratic equation.

2) Assuming that $B_Z = B_T / (2)^{1/2}$ use equations 1 and 4 to find a function that gives Dst in terms of V.

3) From equations 2 and 4, find a function that gives Dst in terms of B_T.

Answer Key:

(1) $Dst = -2.846 + 6.54\ B_z - 0.118B_z^2 - 0.002B_z^3$

(2) $Dst = 0.00052 \times (0.22\ V + 340)^2$

(3) $T = -0.0042 \times V + 5.14$

(4) $B_T = 0.047\ V + 0.644$

Problem 1: From the equation 2 and 3: Dst in terms of the transit time of the CME.

Eqn 3: solve for V. $V = (T - 5.14)/(-0.0042) = -238.1\ T + 1223.8$
Eqn 2: Substitute for V in terms of T:

$$Dst = 0.00052 \times (0.22\ (-238.1\ T + 1223.8) + 340)^2$$
$$= 0.00052 \times (609.2 - 52.4\ T)^2$$

In expanded form: $Dst = 1.4\ T^2 - 33.2\ T + 193.0$ in nT units

Problem 2: Assuming that $B_Z = B_T / (2)^{1/2}$ use equations 1 and 4 and find Dst in terms of V.

Eqn 4: $Bz = B_T / (2)^{1/2} = (0.047\ V + 0.644)/(2)^{1/2}$
$= 0.033\ V + 0.46$

Substituting into Eq 1:

$$Dst = -2.846 + 6.45\ (0.033\ V + 0.46) - 0.118\ (0.033\ V + 0.46)^2 - 0.002\ (0.033\ V + 0.46)^3$$

$$= (-2.846 + 0.46*6.45 - 0.118*0.46^2 - 0.002* 0.46^3) +$$
$$(6.45*0.033 -0.118*2*0.46*0.033 -0.002*3.0* 0.46^2*0.033)\ V +$$
$$(-0.002*3.0*0.46*0.033^2)\ V^2 -0.002 * 0.033^3\ V^3$$

$$Dst = 0.096 + 0.21\ V - 3.0 \times 10^{-6}\ V^2 - 7.2 \times 10^{-8}\ V^3$$

Problem 3: From equations 2 and 4, find a function that gives Dst in terms of B_T.
Eq 4: Solve for V

$$V = (B_T - 0.644)/0.047 = 21.3\ B_T - 13.7$$

Substitute into Eqn 1 : $Dst = 0.00052 \times (0.22\ (21.3\ B_T - 13.7) + 340)^2$
$= 0.00052\ (4.7\ B_T + 337)^2$

Expanded: $Dst = 0.011\ B_T^2 + 1.65\ B_T + 59.1$ in nT units